

① The trisection points, A & B, can be computed by $\vec{A} = \vec{P} + \frac{1}{3}\vec{PQ}$ & $\vec{B} = \vec{P} + \frac{2}{3}\vec{PQ}$, or equivalently using $\vec{Q} + t\vec{QP}$, $t = 1/3$ & $t = 2/3$.

$$\vec{P} + t\vec{PQ} = (8\vec{i} + 5\vec{j}) + t(-7\vec{i} - 12\vec{j})$$

$$t = 1/3: \vec{A} = (8 + \frac{-7}{3})\vec{i} + (5 + \frac{-12}{3})\vec{j} = \frac{17}{3}\vec{i} + \vec{j}, \text{ so } A(\frac{17}{3}, 1)$$

$$t = 2/3: \vec{B} = (8 + \frac{-14}{3})\vec{i} + (5 + \frac{-24}{3})\vec{j} = \frac{10}{3}\vec{i} - 3\vec{j}, \text{ so } B(\frac{10}{3}, -3)$$

② $\vec{W} = 12 \cdot \frac{\vec{v}}{|\vec{v}|} = 12 \cdot \frac{(\vec{i} + 2\vec{j} - 2\vec{k})}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{12\sqrt{5}}{5}\vec{i} + \frac{24\sqrt{5}}{5}\vec{j} - \frac{24\sqrt{5}}{5}\vec{k}$

③a $\vec{A} \cdot \vec{B} = 1 \cdot 3 + 10 \cdot 0 + (-2)(-4) = 11$

③b To find the angle θ between \vec{A} & \vec{B} , solve for θ :

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta = \sqrt{1^2 + 10^2 + 2^2} \cdot \sqrt{3^2 + 4^2} \cos \theta = 11 \text{ (part a)}$$

$$\text{so } \cos \theta = \frac{11}{\sqrt{105} \cdot 5} \Rightarrow \theta = \cos^{-1}\left(\frac{11}{\sqrt{105} \cdot 5}\right) \approx 77.602^\circ$$

③c Vector projection of \vec{B} on \vec{A} : $\left(\frac{\vec{B} \cdot \vec{A}}{|\vec{A}|^2}\right) \vec{A} = \frac{11}{105}(\vec{i} + 10\vec{j} - 2\vec{k})$

$$= \frac{11}{105}\vec{i} + \frac{110}{105}\vec{j} - \frac{22}{105}\vec{k} = \frac{11}{105}\vec{i} + \frac{22}{21}\vec{j} - \frac{22}{105}\vec{k}$$

④ $2x + 3y + 4z + D = 0$, solved by $(x, y, z) = (1, 2, -3)$

$$\Rightarrow 2 + 6 - 12 + D = 0 \Rightarrow D = 4: \quad 2x + 3y + 4z + 4 = 0$$

⑤ $A(2, -2, -1), B(-3, 4, 1), C(4, 2, 3)$. A normal vector to the plane containing A, B, C can be found various ways. One is:

$$\vec{AB} \times \vec{AC} = \vec{N} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 6 & 2 \\ 2 & 4 & 4 \end{vmatrix} = 16\vec{i} + 24\vec{j} - 32\vec{k}$$

More convenient is $\frac{1}{8}\vec{N} = 2\vec{i} + 3\vec{j} - 4\vec{k}$, (same direction)

The plane: $2x + 3y - 4z + D = 0$, solve for D using A or B or C.

using A: $2(2) + 3(-2) - 4(-1) + D = 0 \Rightarrow 2 + D = 0 \Rightarrow D = -2$

plane: $2x + 3y - 4z - 2 = 0$ (or any non-zero multiple of this equation or equivalent equation)

6. Find the angle between the normal vectors to the 2 planes: $\vec{N}_1 = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{N}_2 = 2\vec{i} - 2\vec{j} + \vec{k}$
 If it is acute... done, else use the supplement of the angle if it is obtuse.

$$\theta: |\vec{N}_1| |\vec{N}_2| \cos \theta = (2)(2) + (1)(-2) + (1)(1)$$

$$\Rightarrow \sqrt{6} (3) \cos \theta = 3, \cos \theta = \frac{3}{3\sqrt{6}} = \frac{\sqrt{6}}{6}$$

$$\theta = \cos^{-1} \left(\frac{\sqrt{6}}{6} \right) \approx \boxed{65.905^\circ}$$

7. $d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$, $(x_1, y_1, z_1) = (0, 1, 3)$
 $Ax + By + Cz + D = 0 \iff 2x - y + 2z + 3 = 0$

$$d = \frac{|(2)(0) + (-1)(1) + (2)(3) + 3|}{\sqrt{2^2 + 1^2 + 2^2}} = \boxed{\frac{8}{3}}$$

8a. $(x_1, y_1, z_1) = (4, -3, 5)$, $\vec{N} = A\vec{i} + B\vec{j} + C\vec{k}$
 $= -2\vec{i} + 3\vec{j} + 4\vec{k}$

Symmetric eqns: $\frac{x-x_1}{A} = \frac{y-y_1}{B} = \frac{z-z_1}{C}$

$$\Rightarrow \boxed{\frac{x-4}{-2} = \frac{y+3}{3} = \frac{z-5}{4}}$$

8b. Parametric equations: set each expression in 8a equal to t and solve for the variable:

$$\left. \begin{array}{l} \frac{x-4}{-2} = t \rightarrow x = -2t + 4 \\ \frac{y+3}{3} = t \rightarrow y = 3t - 3 \\ \frac{z-5}{4} = t \rightarrow z = 4t + 5 \end{array} \right\} t \in \mathbb{R}$$

9. $\vec{PQ} = 4\vec{i} + \vec{j} + 8\vec{k} = A\vec{i} + B\vec{j} + C\vec{k}$ $\cos \alpha = \frac{A}{d} = \frac{4}{9}$

$$|\vec{PQ}| = \sqrt{4^2 + 1^2 + 8^2} = 9 = d$$

$$\cos \beta = \frac{B}{d} = \frac{1}{9}$$

$$\cos \gamma = \frac{C}{d} = \frac{8}{9}$$

$$\alpha = \cos^{-1} \left(\frac{4}{9} \right) \approx 63.612^\circ$$

$$\beta = \cos^{-1} \left(\frac{1}{9} \right) \approx 83.621^\circ$$

$$\gamma = \cos^{-1} \left(\frac{8}{9} \right) \approx 27.266^\circ$$

10) $\vec{C} = \vec{A} \times \vec{B}$ is defined by

(a) \vec{C} normal to the plane containing \vec{A} & \vec{B}
(equiv. \vec{C} is perpendicular to both \vec{A} & \vec{B})

(b) $|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$, θ the angle between \vec{A} & \vec{B} .

(c) the "right-hand" orientation. (a) leaves open 2 directions. The right-hand fingers curling from \vec{A} towards \vec{B} will result in the extended thumb to point in the direction of \vec{C} .

11) $\vec{A} = \vec{i} + 10\vec{j} - 2\vec{k}$, $\vec{B} = 3\vec{i} + 0\vec{j} - 4\vec{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 10 & -2 \\ 3 & 0 & -4 \end{vmatrix} = \boxed{-40\vec{i} - 2\vec{j} - 30\vec{k}}$$

12) $A(1, 2, 3)$, $B(2, 12, 1)$, $C(4, 2, -1)$ vertices of ΔABC

If $\vec{V} = \vec{AB} \times \vec{AC}$ then $|\vec{V}|$ is the area of the parallelogram made by 2 copies of ΔABC , so area $\Delta ABC = \frac{1}{2} |\vec{V}|$

$$\vec{V} = (\vec{i} + 10\vec{j} - 2\vec{k}) \times (3\vec{i} + 0\vec{j} - 4\vec{k})$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 10 & -2 \\ 3 & 0 & -4 \end{vmatrix} = -40\vec{i} - 2\vec{j} - 30\vec{k}$$

$$|\vec{V}| = \sqrt{40^2 + 2^2 + 30^2} = \sqrt{2504} = 2\sqrt{626}$$

area of $\Delta ABC = \boxed{\sqrt{626}}$

13) $(3, 4, 1)$

$(0, 0, 0)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{vmatrix} \cdot \frac{1}{\sqrt{14}}$$

$d = |\vec{u} \times \vec{v}|$

$\frac{x}{2} = \frac{y}{3} = \frac{z}{1} \Rightarrow (0, 0, 0)$ is a point on the line.

$2\vec{i} + 3\vec{j} + \vec{k} = \vec{W}$
 \vec{u} = unit vector in direction of \vec{W}
 $= \frac{\vec{W}}{\sqrt{2^2 + 3^2 + 1^2}} = \frac{\vec{W}}{\sqrt{14}} = \frac{1}{\sqrt{14}}(2\vec{i} + 3\vec{j} + \vec{k})$

$\vec{v} = 3\vec{i} + 4\vec{j} + \vec{k}$

$= (-\vec{i} + \vec{j} - \vec{k}) \frac{1}{\sqrt{14}}$, $|\vec{u} \times \vec{v}| = \frac{\sqrt{3}}{\sqrt{14}} = \frac{\sqrt{42}}{14}$

$\frac{\sqrt{42}}{14}$

$\frac{43\sqrt{227}}{227}$

14)

$d = |\vec{P}_1 \vec{P}_2 \cdot \vec{u}|$, \vec{u} a unit vector parallel to $\vec{v}_1 \times \vec{v}_2$, $P_1(2, 1, -1)$, $P_2(4, 0, 5)$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & 3 \\ -1 & 4 & -5 \end{vmatrix} = -22\vec{i} - 18\vec{j} - 10\vec{k}$$

$\vec{u} = \frac{-22\vec{i} - 18\vec{j} - 10\vec{k}}{\sqrt{908}}$

$\vec{P}_1 \vec{P}_2 = 2\vec{i} - \vec{j} + 6\vec{k}$

$|\vec{u} \cdot \vec{P}_1 \vec{P}_2| = \frac{86}{\sqrt{908}} = \frac{86\sqrt{908}}{908}$